SPHERICAL FILM TRANSDUCER IN A THERMOANEMOMETER

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Results are presented on the operation of a spherical film transducer in a thermoanemometer.

A thermoanemometer probe may be used to examine a three-dimensional flow of gas or liquid, this being a metal film on an insulating substrate. Heat passes from the heated film element into the flow and into the substrate. To reduce the heat leak from the substrate and minimize the effects on the metrological characteristics it is necessary for the surface heat source to be bounded by a closed volume. This requirement is met by a probe taking the form of a sphere with six insulated film resistors placed perpendicular to the three Cartesian coordinate axes 1,1'; 2,2'; 3,3' (Fig. 1). The resistors are connected into the measurement circuit and represent the arms of six bridges. The operation is based on the local heat transfer to the flow from the heated sphere, this being larger in the forward part than in the rear one [1]. The directional characteristics (Fig. 2) show that the transducer enables one to measure the components of the velocity vector for three-dimensional flow within an angular region of 360°.

To examine the effects of the substrate on the metrological characteristics, we consider a model constituted by a sphere of radius R having a surface heat source of thichness h ($h \ll R$). the temperature is taken as constant over the thickness of the source and equal to the surface temperature of the spherical substrate. The metal film is of high thermal conductivity, so the temperatures of the individual zones in the metal are equal and there is a nearly symmetrical temperature pattern within the spherical substrate. In the steady state, the temperature of the latter does not vary with radius, so there is no temperature gradient within the probe and there is no heat transfer from the heated film component into the substrate, with the overall heat-transfer coefficient for the film element equal to the convective value.

We now consider the operation in the unsteady state, which occurs with a turbulent flow. We assume that the fluctuating component of the film temperature is constant over the thickness and is equal to the fluctuating component of the substrate surface temperature, and also that $t(r,\tau)|_{r=R} \ll T(r)|_{r=R}$, $\alpha(\tau) \ll \alpha$. The following is the equation of thermal conduction for the spherical substrate in the unsteady state [2]:

$$\frac{\partial^2 t(r,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial t(r,\tau)}{\partial r} = \frac{1}{a}\frac{\partial t(r,\tau)}{\partial \tau}. \quad (1)$$

The boundary conditions are

$$-\lambda \frac{\partial t(r,\tau)}{\partial r}\Big|_{r=R} + g(\tau) - [\alpha + \alpha(\tau)][T(r)|_{r=R} + t(r,\tau)|_{r=R}] = 0, \quad (2)$$

$$t(r,\tau)|_{r=R_0} = 0. \quad (3)$$

The Laplace transformation with respect to τ:

$$\frac{d^2[rt(r,s)]}{dr^2} - \frac{s}{a} rt(r,s) = 0. \quad (4)$$

The boundary conditions are

$$\alpha(s)T(r)|_{r=R} + \alpha t(r,s) + \lambda\frac{dt(r,s)}{dr} = g(s), \quad (5)$$

$$t(r,s)|_{r=R_0} = 0. \quad (6)$$

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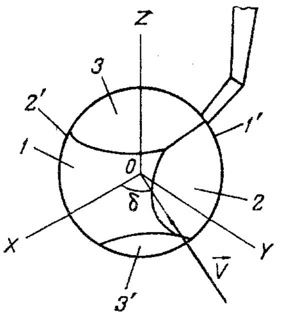

Fig. 1

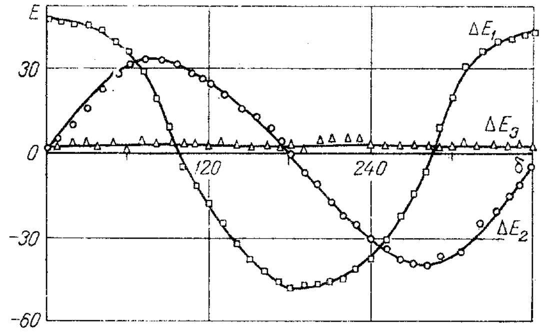

Fig. 2

Fig. 1 Thermoanemometer film spherical probe.

Fig. 2. Dependence of output signal E (mV) on angle δ (deg).

The solution to (4) with (5) and (6) is

$$t(r,\ s)=\frac{1}{r}\left[g(s)-\alpha(s)T(r)|_{r=R}\right]\left[\operatorname{ch}\left(r\sqrt{\frac{s}{a}}\right)-\operatorname{sh}\left(r\sqrt{\frac{s}{a}}\right)\times\right.$$
$$\left.\times\operatorname{cth}\left(R_0\sqrt{\frac{s}{a}}\right)\right]\left[\operatorname{ch}\left(\sqrt{\frac{s}{a}}\right)-\operatorname{cth}\left(R_0\sqrt{\frac{s}{a}}\right)\operatorname{sh}\left(\sqrt{\frac{s}{a}}\right)\right]^{-1}\alpha^{-1}+$$
$$+\frac{1}{r}\left[g(s)-\alpha(s)T(r)|_{r=R}\right]\left[\operatorname{ch}\left(r\sqrt{\frac{s}{a}}\right)-\operatorname{sh}\left(r\sqrt{\frac{s}{a}}\right)\times\right.$$
$$\left.\times\operatorname{cth}\left(R_0\sqrt{\frac{s}{a}}\right)\right]\left\{\sqrt{\frac{s}{a}}\operatorname{sh}\left(\sqrt{\frac{s}{a}}\right)-\operatorname{ch}\left(\sqrt{\frac{s}{a}}\right)-\right.$$
$$\left.-\operatorname{cth}\left(R_0\sqrt{\frac{s}{a}}\right)\left[\sqrt{\frac{s}{a}}\operatorname{ch}\left(\sqrt{\frac{s}{a}}\right)-\operatorname{sh}\left(\sqrt{\frac{s}{a}}\right)\right]\right\}^{-1}\mathrm{Bi}. \tag{7}$$

The Laplace transform of the fluctuations in heat production by the surface source is defined by

$$g(s)=bt(r,\ s)|_{r=R}\,. \tag{8}$$

As with $t(r,\ s)|_{r=R} \ll T(r)|_{r=R}$ and $\alpha(\tau) \ll \alpha$ the fluctuations in the convective heat-transfer coefficient are proportional to the velocity fluctuations, we have from (7) and (8) the transfer function for the spherical transducer as regards velocity fluctuations:

$$W=\frac{g}{\alpha^2}\left(\frac{NL+PM}{N^2+P^2}+i\,\frac{MN-PL}{N^2+P^2}\right), \tag{9}$$

where

$$L=\frac{1}{2\sqrt{2}}\frac{R_0}{R}\frac{\omega}{a}\sqrt{\frac{\omega}{a}}\left[\frac{\omega R_0}{6aR}\left(1-\frac{R_0}{R}\right)+\frac{R_0}{R}\left(1-\frac{R_0}{3R}\right)-\frac{2}{3}\right]; \tag{10}$$

$$M=\frac{1}{2\sqrt{2}}\frac{R_0}{R}\frac{\omega}{a}\sqrt{\frac{\omega}{a}}\left[\frac{\omega R_0}{6aR}\left(1-\frac{R_0}{R}\right)\right.$$
$$\left.-\frac{R_0}{R}\left(1-\frac{R_0}{3R}\right)+\frac{2}{3}\right]; \tag{11}$$

$$N=\frac{1}{2\sqrt{2}}\left(\frac{B}{\alpha}-1\right)\frac{R_0}{R}\frac{\omega}{a}\sqrt{\frac{\omega}{a}}\left[\frac{\omega R_0}{6aR}\left(1-\frac{R_0}{R}\right)+\right.$$
$$\left.+\frac{R_0}{R}\left(1-\frac{R_0}{3R}\right)-\frac{2}{3}\right]+\frac{1}{\mathrm{Bi}}\left[\frac{\omega^2}{6a^2}\left(1-\frac{17}{18}\frac{R_0}{R}\right)+\frac{2R_0}{R}+1\right]; \tag{12}$$

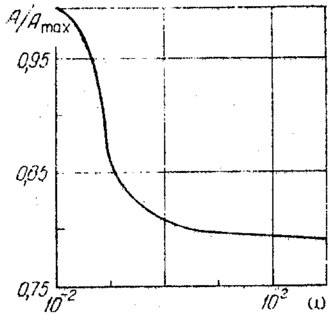

Fig. 3. Frequency response.

$$P = \frac{1}{2\sqrt{2}}\left(\frac{B}{\alpha}-1\right)\frac{R_0}{R}\frac{\omega}{a}\sqrt{\frac{\omega}{a}}\left[\frac{\omega R_0}{6aR}\left(1-\frac{R_0}{R}\right)-\frac{R_0}{R}\left(1-\frac{R_0}{3R}\right)+\frac{2}{3}\right]-\frac{\omega}{2a\mathrm{Bi}}\left(1+\frac{R_0}{3R}\right). \quad (13)$$

From (9), the frequency response takes the following form (Fig. 3):

$$A = \left[\left(\frac{g}{\alpha^2}\frac{NL+PM}{N^2+P^2}\right)^2 + \left(\frac{g}{\alpha^2}\frac{MN-PL}{N^2+P^2}\right)^2\right]^{0,5}. \quad (14)$$

The substrate does not influence the static characteristics in the stationary state, but it does have an effect on the dynamic characteristics in the unsteady state. One can reduce the effects of the substrate on the dynamic characteristics by making it hollow, which is technologically complicated and makes the transducer mechanically weak. The absence of temperature gradients along the radius means that one cannot use internal heating, which would improve the dynamic characteristics of the film transducers. It is necessary to use a material of low thermal conductivity to minimize the effects of the substrate.

NOTATION

T(r), probe temperature, °K; T, flow temperature, °K; t(r,τ), fluctuating component of probe temperature; R, probe radius, m; R_0, thermal-wave attenuation radius, m; a, thermal diffusivity, m/sec; α, convective heat-transfer coefficient, W/m•K; α(τ), fluctuating component of the heat-transfer coefficient, W/m•K; λ, thermal conductivity, W/m•K; g, heat released per unit time per unit surface, J/sec•m; g(τ), heat release fluctuation of surface source, J/sec•m; s, Laplace transform parameter; τ, time, sec; ω, frequency of fluctuations, Hz.

LITERATURE CITED

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